

(4) $\frac{2}{3}$

(5) $\frac{\pi}{2}$

2 (1) 0

(2) π

練習問題

1 (1) $\frac{17}{2}$

(2) $2 + \log 5$

(3) $\frac{1}{2} \log \frac{1}{3}$

(4) $\frac{4}{3}$

(5) $1 - \frac{\pi}{4}$

(6) $\frac{3}{4}\pi$

2 (1) 0

(2) 0

置換積分(1)

例題

C は積分定数とする.

(1) $\frac{2}{15} (3x - 2)(x + 1)\sqrt{x + 1} + C$

(2) $\frac{1}{3}e^{3x} - \frac{1}{2}e^{2x} + e^x - \log(e^x + 1) + C$

練習問題

C は積分定数とする.

(1) $-\frac{2}{15} (3x + 2)(1 - x)\sqrt{1 - x} + C$

(2) $\frac{2}{27} (3x - 2)\sqrt{3x + 1} + C$

(3) $\log \frac{e^x}{e^x + 1} + C$

(4) $\frac{1}{2}e^{2x} - \log(e^{2x} + 2) + C$

置換積分(2)

例題

1 (1) $\frac{4}{15}$

(2) $\log \frac{2e}{e + 1}$

2 (1) $\sin x = \frac{2t}{1 + t^2}, \cos x = \frac{1 - t^2}{1 + t^2}$

(2) $\frac{2}{1 + t^2}$

(3) $\log \left| 1 + \tan \frac{x}{2} \right| + C$ (C は積分定数)

(4) $\log 2$

練習問題

1 (1) $-\frac{4}{15}$

(2) $\frac{10}{3}$

(3) $\frac{1}{2}(e^2 - 1) - \log \frac{e^2 + 2}{3}$

2 (1) $\frac{2t}{1 + t^2}$

(2) $\frac{2}{1 + t^2}$

(3) $\log \left| \tan \frac{x}{2} \right| + C$

(4) $\frac{1}{2} \log 3$

置換積分(3)

例題

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{12}$

(3) $\frac{\sqrt{3}}{18}\pi$

練習問題

(1) $\frac{2}{3}\pi + \sqrt{3}$

(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{8} + \frac{1}{4}$

部分積分(1)

例題

C は積分定数とする.

[1] (1) $-x \cos x + \sin x + C$

(2) $xe^x - e^x + C$

(3) $\frac{1}{3}x^3 \log x - \frac{1}{9}x^3 + C$

(4) $\frac{1}{2}(1+2x) \log(1+2x) - x + C$

[2] (1) $x^2 e^x - 2xe^x + 2e^x + C$

(2) $x(\log x)^2 - 2x \log x + 2x + C$

練習問題

C は積分定数とする.

[1] (1) $x \sin x + \cos x + C$

(2) $-xe^{-x} - e^{-x} + C$

(3) $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$

(4) $-(1-x) \log(1-x) - x + C$

[2] (1) $-x^2 \cos x + 2x \sin x + 2 \cos x + C$

(2) $\frac{1}{2}x^2 (\log x)^2 - \frac{1}{2}x^2 \log x + \frac{1}{4}x^2 + C$

部分積分(2)

例題

[1] (1) $\frac{\pi}{2} - 1$

(2) $\frac{1}{4}$

(3) $e - 2$

(4) $\frac{e^2 + 1}{4}$

(5) 1

[2] $-\frac{\pi^2}{2}$

[3] $\frac{\pi^2}{4}$

練習問題

[1] (1) π

(2) $\frac{\sqrt{2}}{24}\pi + \frac{\sqrt{2}}{18}$

(3) $\frac{1}{9}$

(4) $\frac{2}{9}e^3 + \frac{1}{9}$

(5) $11 \log 11 - 10$

[2] $e - 2$

[3] $\frac{\pi^2}{4}$

部分積分(3)

例題

[1] (1) 1

(2) $I_{n+1} = e - (n+1)I_n$

(3) $-2e + 6$

[2] (1) $I_0 = \frac{\pi}{2}, I_1 = 1$

(2) $I_{n+2} = \frac{n+1}{n+2}I_n$

(3) $\frac{8}{15}$

練習問題

[1] (1) 1

(2) $I_{n+1} = e - (n+1)I_n$

(3) $-2e + 6$

[2] (1) $I_0 = \frac{\pi}{4}, I_1 = \frac{1}{2} \log 2$

(2) $I_{n+2} = \frac{1}{n+1} - I_n$

(3) $-\frac{2}{3} + \frac{\pi}{4}$

絶対値を含む関数の定積分

例題

(1) $2\sqrt{e} - \frac{1}{2}e - \frac{3}{2}$

(2) $4 \log 2 + e - 5$

(3) $2\pi + 1$

練習問題

(1) 1

(2) $\frac{3\sqrt{3}}{2}$

(3) $\frac{e^2}{4} + \frac{1}{2} - \frac{3}{4e^2}$

偶関数・奇関数の定積分

例題

(1) $-\pi$

(2) 0

(3) $3 \log_2 6$

練習問題

(1) 0

(2) 2π

(3) $\frac{4}{3}$

(4) $2(\sqrt{2} - 1)$

定積分で表された関数(1)

例題

[1] (1) $\frac{1}{1+x^2}$

(2) $-\sin x$

(3) $2x^{11}(x^2+1)^2$

[2] (1) $\frac{e}{2}$

(2) $\log 2x$

練習問題

[1] (1) e^{2x}

(2) $-\cos x + 1$

(3) $-\frac{1}{x^2+1}$

(4) $\log(x+1) - \log x$

[2] (1) $a = 2$

(2) $\frac{2 \log x}{x} + 8$

定積分で表された関数(2)

例題

(1) $\sin^2 x + \frac{\pi}{4-2\pi}$

(2) $3x - e^x + e - 3$

練習問題

(1) $\sin x + \frac{2}{1-\pi}$

(2) $\sin x - \frac{\pi}{8}$

定積分の種々の問題(1)

例題

(1) $\frac{3}{4}$

(2) $\frac{1}{2} \log 2$

(3) π

練習問題

(1) $\frac{2}{3}$

(2) $\log 2$

(3) $\frac{\pi}{4}$

定積分の種々の問題(2)

例題

(1) 略

(2) 略

練習問題

(1) 略

(2) 略

面積(1)

例題

[1] (1) 1

(2) $\frac{24}{\log 3} - e^3 + e$

(3) $1 - \log 2$

(4) $\log 2$

2 $\frac{e^2}{4} - \frac{e}{2} - \frac{1}{4}$

練習問題

1 (1) 2

(2) $2 \log 2 - 1$

(3) $\frac{14}{3}$

(4) $\frac{3}{2} - 2 \log 2$

2 $\frac{1}{2}e - \frac{1}{2e} - 1$

面積(2)

例題

(1) $\frac{8}{3}$

(2) $\frac{1}{3}$

(3) $\frac{1}{2} \log 2$

練習問題

(1) $e^2 - 1$

(2) 1

(3) $\frac{9}{2}$

(4) 2

面積(3)

例題

1 6π

2 4π

練習問題

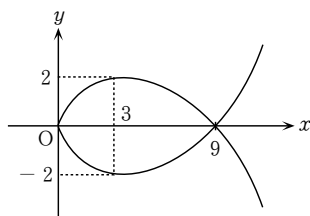
1 πab

2 π

面積(4)

例題

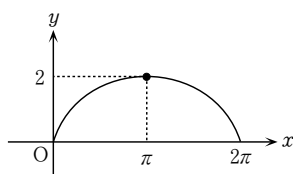
(1)



(2) $\frac{72\sqrt{3}}{5}$

練習問題

(1)



(2) 3π

体積(1)

例題

(1) $\frac{\pi^2}{4}$

(2) $\pi(e - 2)$

練習問題

(1) $\frac{\pi}{2}(e^2 - 1)$

(2) $\frac{\pi^2}{8} - \frac{\pi}{4}$

(3) $\frac{16}{15}\pi$

体積(2)

例題

1 $\frac{2}{3}\pi$

2 $\frac{\pi}{3}h^2(3r - h)$

練習問題

1 $\frac{4}{3}\pi r^3$